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RELATIONSHIP BETWEEN THE AERODYNAMIC DAMPING DERIVATIVES MEASURED AS A FUNCTION OF INSTANTANEOUS ANGULAR DISPLACEMENT AND THE AERODYNAMIC DAMPING DERIVATIVES MEASURED AS A FUNCTION OF OSCILLATION AMPLITUDE

by Bass Redd, Dennis M. Olsen, and Richard L. Barton

Manned Spacecraft Center
Houston, Texas

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SUMMARY

A method is presented which relates the aerodynamic damping derivatives as a function of instantaneous angular displacement with the aerodynamic derivatives as a function of oscillation amplitude. Each derivative is expressed in a power series, and by applying the Kryloff-Bogoliuboff equivalent linearization technique, the solution is obtained by equating like coefficients. It is assumed that the damping per cycle is small, the damping coefficient is symmetrical, and the pitching moment is linear. Comparisons with numerical integration results show no appreciable error.

The effect of a nonlinear pitching moment is considered for a representative case and is found to be small except when approaching unstable trim.

INTRODUCTION

The aerodynamic pitch damping coefficient can be obtained from wind-tunnel tests by two general methods. In one method, an example of which is small-amplitude forced-oscillation tests (ref. 1), the aerodynamic pitch damping coefficient $C_{m_q} + C_{m_{\dot{\alpha}}}$ is measured as a function of instantaneous angular displacement θ by forced oscillations of small amplitudes at discreet angles of attack. In the other method, examples of which are ballistic, free-flight, or free-oscillation wind-tunnel tests (ref. 2), the average aerodynamic pitch damping coefficient $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$ is measured as a function of oscillation peak amplitude θ_0 by determining the change in peak amplitude per cycle.

To be used in current digital trajectory computer programs the aerodynamic pitch damping coefficient must be expressed as a function of instantaneous angular displacement rather than oscillation peak amplitude. Since ballistic, free-flight, and free-oscillation wind-tunnel experiments determine the damping as a function of oscillation peak amplitude, a method of expressing these results as a function of instantaneous angle of attack is needed. It is the purpose of this paper to develop a method whereby damping as a function of oscillation amplitude can be expressed as a function of instantaneous angle of attack. The method has the added capability of determining the damping as a function of oscillation amplitude when damping as a function of instantaneous angle of attack is known. This latter property allows an oscillation-amplitude time history to be constructed without resorting to numerical integration of the equation of motion.

SYMBOLS

A, A_2, A_4, A_6, A_8	arbitrary constants in the series expansion of $f(\theta)$
B, B_2	arbitrary constants in the cubic series for C_m
C, C_2, C_4, C_6, C_8	arbitrary constants in the series expansion of $f(\theta_0)$
C_m	pitching-moment coefficient, $\frac{\text{Pitching moment}}{q_\infty SD}$
C_{m_α}	$\frac{\partial C_m}{\partial \alpha}$
$C_{m_q} + C_{m_{\dot\alpha}}$	damping in pitch coefficient, $\left(\frac{\partial C_m}{\partial \frac{qD}{2V}}\right) + \left(\frac{\partial C_m}{\partial \frac{\dot\alpha D}{2V}}\right)$
$C_{\overline{m}_q} + C_{\overline{m}_{\dot\alpha}}$	average damping in pitch coefficient over one full oscillation
D	reference length
$f(\theta)$	viscous damping as a function of θ
$f(\theta_0)$	viscous damping as a function of θ_0
I	mass moment of inertia
k	slope of assumed linear pitching moment
P	power per cycle as a function of $f(\theta)$
P_0	power per cycle as a function of $f(\theta_0)$
q	angular pitching velocity
q_∞	dynamic pressure

S	reference area
T	period
t	time
V	free-stream velocity
α	angle of attack
$\dot{\alpha}$	rate of change of angle of attack
θ	instantaneous angular displacement
θ_0	oscillation peak amplitude
θ_{lc}	limit-cycle amplitude, linear pitching moment
θ_{lc}^*	limit-cycle amplitude, cubic pitching moment
θ_{ut}	unstable trim amplitude, cubic pitching moment
$\dot{\theta}$	angular velocity
$\ddot{\theta}$	angular acceleration
θ_N	arbitrary angular displacement
ω	angular frequency, $\frac{2\pi}{T}$

ANALYSIS

Derivation of Equations

The second-order differential equation of motion for a single-degree-of-freedom oscillating system with viscous damping proportional to displacement is

$$I\ddot{\theta} + f(\theta)\dot{\theta} + k\theta = 0 \tag{1}$$

where $f(\theta)$ is the instantaneous viscous damping and k is the slope of an assumed linear pitching moment. The function $f(\theta)$ corresponds to the damping coefficient as found in a forced oscillation experiment and, for a vehicle symmetrical in the pitch plane, can be expressed as

$$f(\theta) = A + A_2\theta^2 + A_4\theta^4 + A_6\theta^6 + A_8\theta^8 + \ldots \tag{2}$$

If, instead of small-amplitude oscillations about a fixed mean angle of attack, the vehicle had oscillated harmonically about the trim angle with a large amplitude, the power per cycle would be

$$P = \int_0^T f(\theta)\dot{\theta}^2 dt = \int_0^T \left(A + A_2\theta^2 + A_4\theta^4 + A_6\theta^6 + A_8\theta^8 + \ldots\right)\dot{\theta}^2 dt \tag{3}$$

For a free oscillation, the equation of motion is

$$I\ddot{\theta} + f\left(\theta_0\right)\dot{\theta} + k\theta = 0 \tag{4}$$

where $f\left(\theta_0\right)$ is the viscous damping as a function of oscillation amplitude. The function $f\left(\theta_0\right)$ corresponds to the average damping coefficient as determined in a free-oscillation experiment, and for a vehicle symmetrical in the pitch plane, can be expressed as

$$f\left(\theta_0\right) = C + C_2\theta_0^2 + C_4\theta_0^4 + C_6\theta_0^6 + C_8\theta_0^8 + \ldots \tag{5}$$

The power per cycle can be written as

$$P_0 = f\left(\theta_0\right)\int_0^T \dot{\theta}^2 dt = \left(C + C_2\theta_0^2 + C_4\theta_0^4 + C_6\theta_0^6 + C_8\theta_0^8 + \ldots\right)\int_0^T \dot{\theta}^2 dt \tag{6}$$

Kryloff and Bogoliuboff (ref. 3) have shown that equation (4) is the equivalent linear differential equation of the nonlinear equation of motion, equation (1). As in reference 4, where the power per cycle is assumed to be the same over a given oscillation, then

$$\int_0^T f(\theta)\dot{\theta}^2 dt = f\left(\theta_0\right)\int_0^T \dot{\theta}^2 dt \tag{7}$$

Solving for $f\left(\theta_0\right)$

$$f\left(\theta_0\right) = \frac{\int_0^T f(\theta)\dot{\theta}^2 dt}{\int_0^T \dot{\theta}^2 dt} \tag{8}$$

or its equivalent

$$C + C_2\theta_0^2 + C_4\theta_0^4 + C_6\theta_0^6 + C_8\theta_0^8 + \ldots = \frac{\int_0^T\left(A + A_2\theta^2 + A_4\theta^4 + A_6\theta^6 + A_8\theta^8 + \ldots\right)\dot{\theta}^2 dt}{\int_0^T \dot{\theta}^2 dt} \tag{9}$$

For almost harmonic motion θ can be replaced by

$$\theta = \theta_0 \cos \omega t \tag{10}$$

If the damping moment is small, then θ will be a slowly varying function of time during the period T and as a first approximation can be considered constant over a cycle. The relation of equation (10) allows a straightforward integration of equation (9). Integrating and carrying out the indicated division

$$C + C_2\theta_0^2 + C_4\theta_0^4 + C_6\theta_0^6 + C_8\theta_0^8 = A + \frac{1}{4}A_2\theta_0^2 + \frac{1}{8}A_4\theta_0^4 + \frac{5}{64}A_6\theta_0^6 + \frac{7}{128}A_8\theta_0^8 \tag{11}$$

Equating like coefficients

$$\left.\begin{aligned} C &= A \\ C_2 &= \frac{1}{4}A_2 \\ C_4 &= \frac{1}{8}A_4 \\ C_6 &= \frac{5}{64}A_6 \\ C_8 &= \frac{7}{128}A_8 \end{aligned}\right\} \tag{12}$$

Terms beyond the eighth order were not considered.

If, from a forced-oscillation test, $f(\theta)$ is known in a power series such as equation (2), then $f(\theta_0)$ can be determined directly as

$$f(\theta_0) = A + \frac{1}{4}A_2\theta_0^2 + \frac{1}{8}A_4\theta_0^4 + \frac{5}{64}A_6\theta_0^6 + \frac{7}{128}A_8\theta_0^8 \tag{13}$$

On the other hand, if $f(\theta_0)$ is known from a free-oscillation test and can be expressed in a power series such as equation (5), $f(\theta)$ can be found directly as

$$f(\theta) = C + 4C_2\theta^2 + 8C_4\theta^4 + \frac{64}{5}C_6\theta^6 + \frac{128}{7}C_8\theta^8 \tag{14}$$

Since

$$\left.\begin{aligned} f(\theta) &= \frac{q_\infty SD^2}{2V}\left(C_{m_q} + C_{m_{\dot{\alpha}}}\right) \\ \text{and} \qquad & \\ f\left(\theta_0\right) &= \frac{q_\infty SD^2}{2V}\left(C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}\right) \end{aligned}\right\} \tag{15}$$

the desired relationship between $C_{m_q} + C_{m_{\dot{\alpha}}}$ and $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$ is found.

Methods of Application

<u>Determination of $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$ from measured values of $C_{m_q} + C_{m_{\dot{\alpha}}}$</u>.- If values of $C_{m_q} + C_{m_{\dot{\alpha}}}$ as a function of angular displacement θ have been measured in small-amplitude forced-oscillation tests, the damping coefficient can be expressed in a power series

$$C_{m_q} + C_{m_{\dot{\alpha}}} = \frac{2V}{q_\infty SD^2}\left(A + A_2\theta^2 + A_4\theta^4 + A_6\theta^6 + A_8\theta^8\right)$$

Then, from equation (13) the damping coefficient as a function of amplitude of oscillation is

$$C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}} = \frac{2V}{q_\infty SD^2}\left(A + \frac{1}{4}A_2\theta_0^2 + \frac{1}{8}A_4\theta_0^4 + \frac{5}{64}A_6\theta_0^6 + \frac{7}{128}A_8\theta_0^8\right)$$

With this calculated $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$ the oscillation-amplitude time history can be determined by using the logarithmic decrement relation

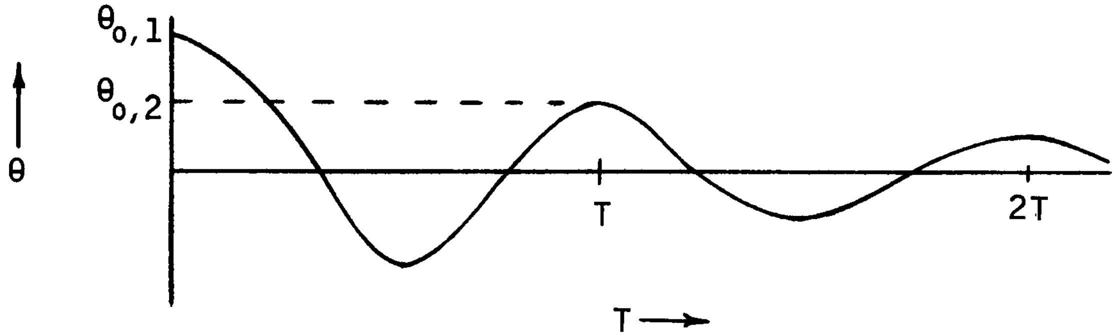
$$\theta_{0,2} = \theta_{0,1}\, e^{\frac{\Delta t}{2I}f\left(\theta_0\right)} = \theta_{0,1} e^{\frac{\Delta t}{2I}\frac{q_\infty SD^2}{2V}\left(C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}\right)} \tag{16}$$

If the measured values of $C_{m_q} + C_{m_{\dot{\alpha}}}$ show negative damping at small amplitudes and positive damping at larger amplitudes, a limit cycle is indicated.

The amplitude of this limit cycle is the amplitude at which the oscillation-amplitude time history, determined by using equation (16), levels out. The amplitude of the limit cycle can also be found by solving for the roots of the power series representation of $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$. The two real roots, opposite in sign but equal in magnitude, near trim will be the amplitude of the limit cycle.

Determination of $C_{m_q} + C_{m_{\dot{\alpha}}}$ from measured values of $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$.- If an oscillation-amplitude time history such as that shown in sketch (a) is obtained from a free-oscillation test, the damping as a function of oscillation amplitude can be calculated with the following equation:

$$f(\theta_0) = \frac{2I}{T} \log_e \frac{\theta_{0,2}}{\theta_{0,1}} \qquad (17)$$



Sketch (a)

Then, $f(\theta_0)$ can be plotted against θ_0 and expressed in the power series of equation (5). Damping as a function of angular displacement can be calculated from equation (14). From equation (15), the damping coefficient as a function of angular displacement is

$$C_{m_q} + C_{m_{\dot{\alpha}}} = \frac{2V}{q_\infty SD^2} f(\theta)$$

Numerical Comparisons

The following two theoretical methods have been presented to relate the pitch damping derivatives as a function of angular displacement with the pitch damping derivatives as a function of oscillation amplitude:

(1) Determination of $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$ from measured values of $C_{m_q} + C_{m_{\dot{\alpha}}}$

(2) Determination of $C_{m_q} + C_{m_{\dot{\alpha}}}$ from measured values of $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$

In order to ascertain the accuracy of these methods, arbitrary damping derivative coefficients $C_{m_q} + C_{m_{\dot{\alpha}}}$ were programed in the single-degree-of-freedom equation of motion

$$I\ddot{\theta} - \frac{q_\infty SD^2}{2V}\left(C_{m_q} + C_{m_{\dot{\alpha}}}\right)\dot{\theta} - q_\infty SDC_{m_\alpha}\theta = 0 \tag{18}$$

and numerically integrated. The oscillation-amplitude time history obtained was used to calculate $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$. The results of these calculations were then compared with the theoretical solution to verify method 1. To ascertain the validity of method 2, $C_{m_q} + C_{m_{\dot{\alpha}}}$ was obtained from the calculated values of $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$ and then were compared with the input values.

The following two cases present numerical comparisons made to verify the two methods.

Case A.- The first case is that of a quadratic with positive damping. The value of $C_{m_q} + C_{m_{\dot{\alpha}}}$ was chosen as $-6.55 \times 10^{-3}\ \theta^2$. The oscillation-amplitude time history in figure 1 was obtained by numerical integration of equation (18); and from the time history, $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$ was calculated by using equation (17). The theoretical damping coefficient was then determined by using method 1. The calculated and theoretical values are compared in figure 2.

Method 2 was used to obtain $C_{m_q} + C_{m_{\dot{\alpha}}}$ from the calculated values of $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$ of figure 2, and the results are compared with the actual value $-6.55 \times 10^{-3}\ \theta^2$ in figure 3.

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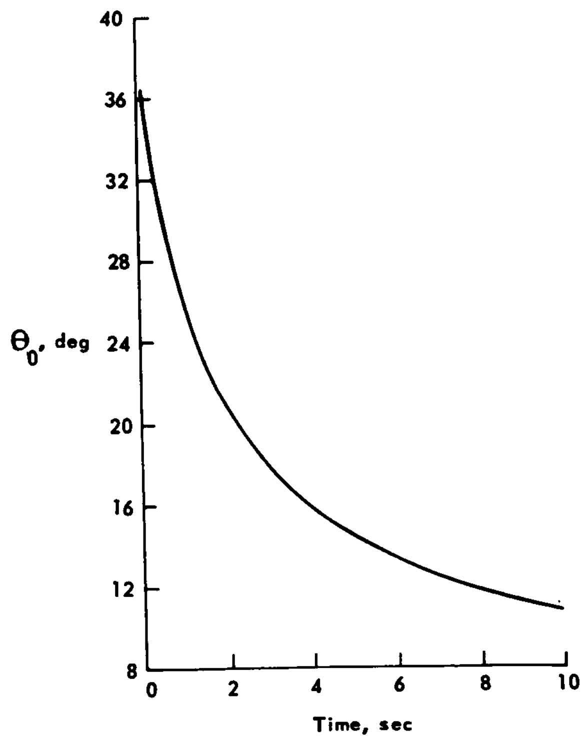

Figure 1. - Oscillation-amplitude time history for case A, $C_{m_q} + C_{m_{\dot{\alpha}}} = -6.55 \times 10^{-3}\theta^2$.

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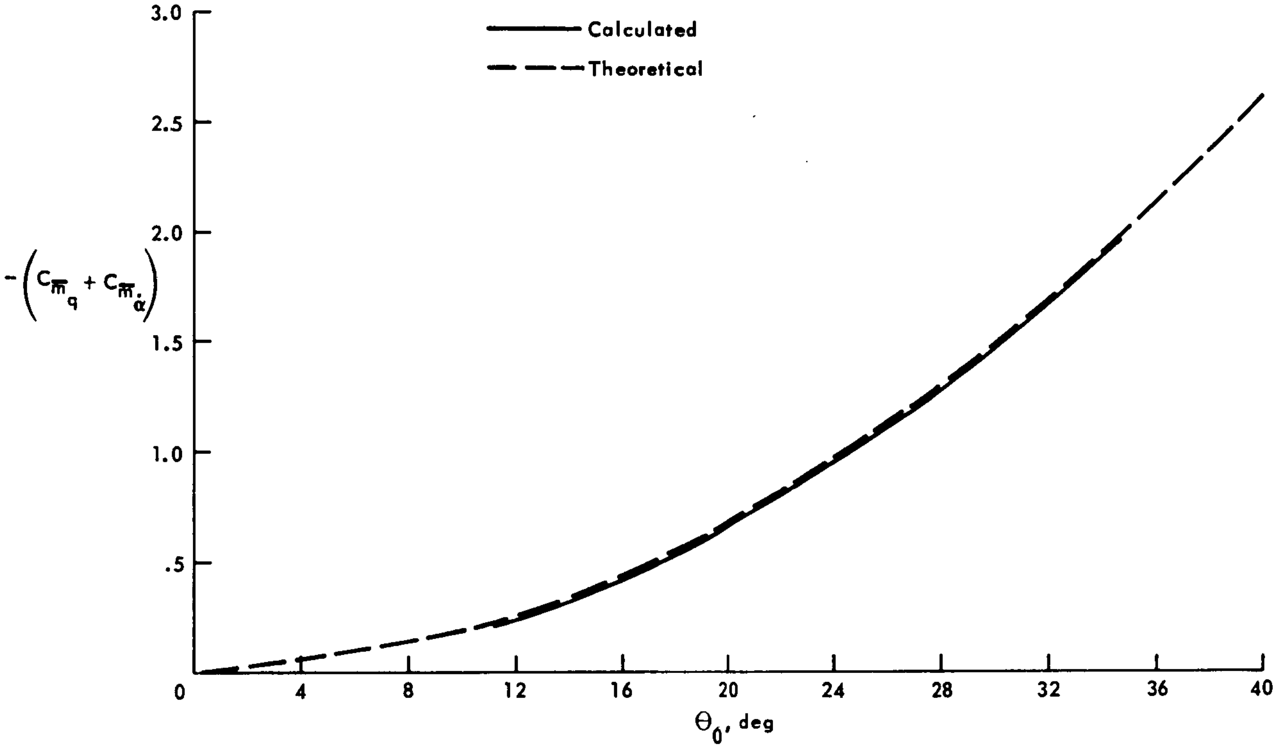

Figure 2. - Variation of $C_{\bar{m}_q} + C_{\bar{m}_{\dot{\alpha}}}$ with Θ_0 for case A.

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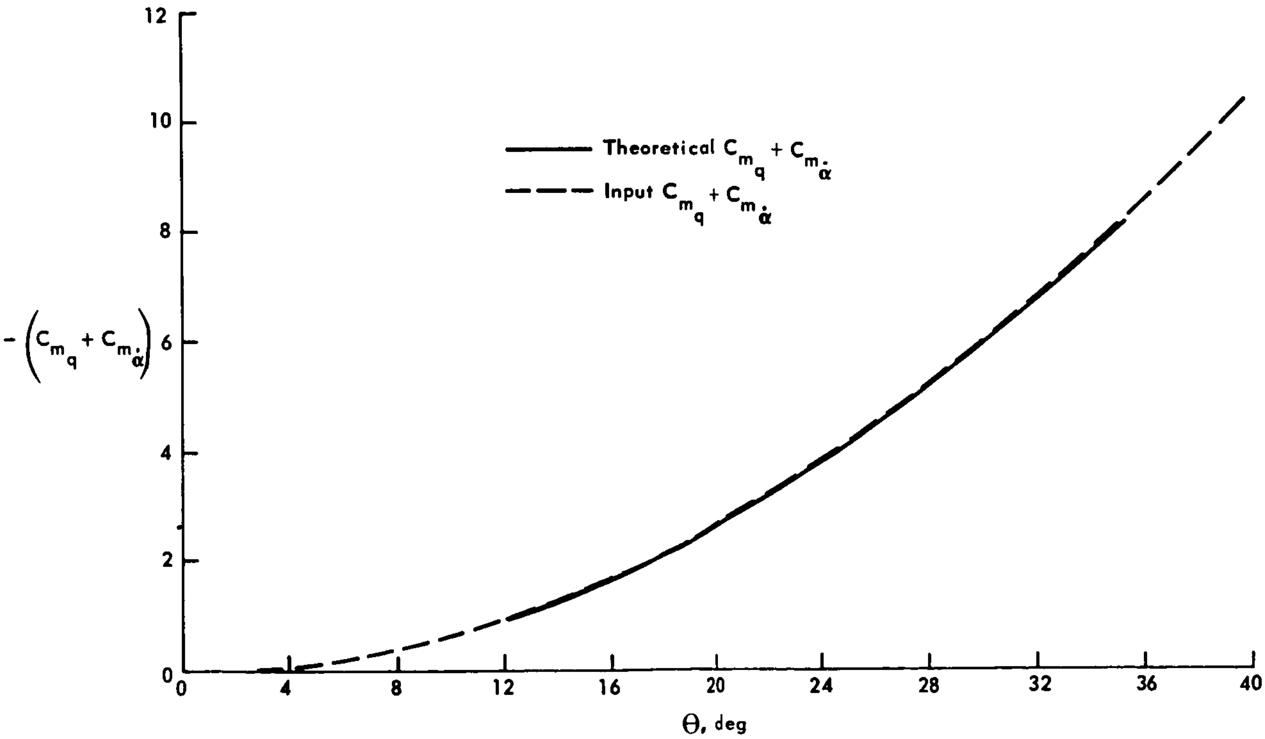

Figure 3. - Variation of $C_{m_q} + C_{m_{\dot{\alpha}}}$ with Θ for case A.

Case B.- The second case is that of a quadratic with both positive and negative damping. The value of $C_{m_q} + C_{m_{\dot{\alpha}}}$ was chosen as $2.0 - 6.55 \times 10^{-3} \theta^2$ Equation (18) was numerically integrated to obtain the oscillation-amplitude time history of figure 4 which shows a limit cycle at 35°. From figure 4, $C_{\bar{m}_q} + C_{\bar{m}_{\dot{\alpha}}}$ was calculated by using equation (16), and the results are compared in figure 5 with the theoretical coefficients obtained from method 1. Note that both the theoretical and computed values show zero damping at $\theta_0 = 35°$, as would be expected.

Method 2 was used to obtain $C_{m_q} + C_{m_{\dot{\alpha}}}$ from the calculated values of $C_{\bar{m}_q} + C_{\bar{m}_{\dot{\alpha}}}$, and the results are compared with the actual coefficient $2.0 - 6.55 \times 10^{-3} \theta^2$ in figure 6. Both solutions have damping coefficients of 0 at $\theta = 17.5°$, which is exactly one-half the amplitude of the limit cycle.

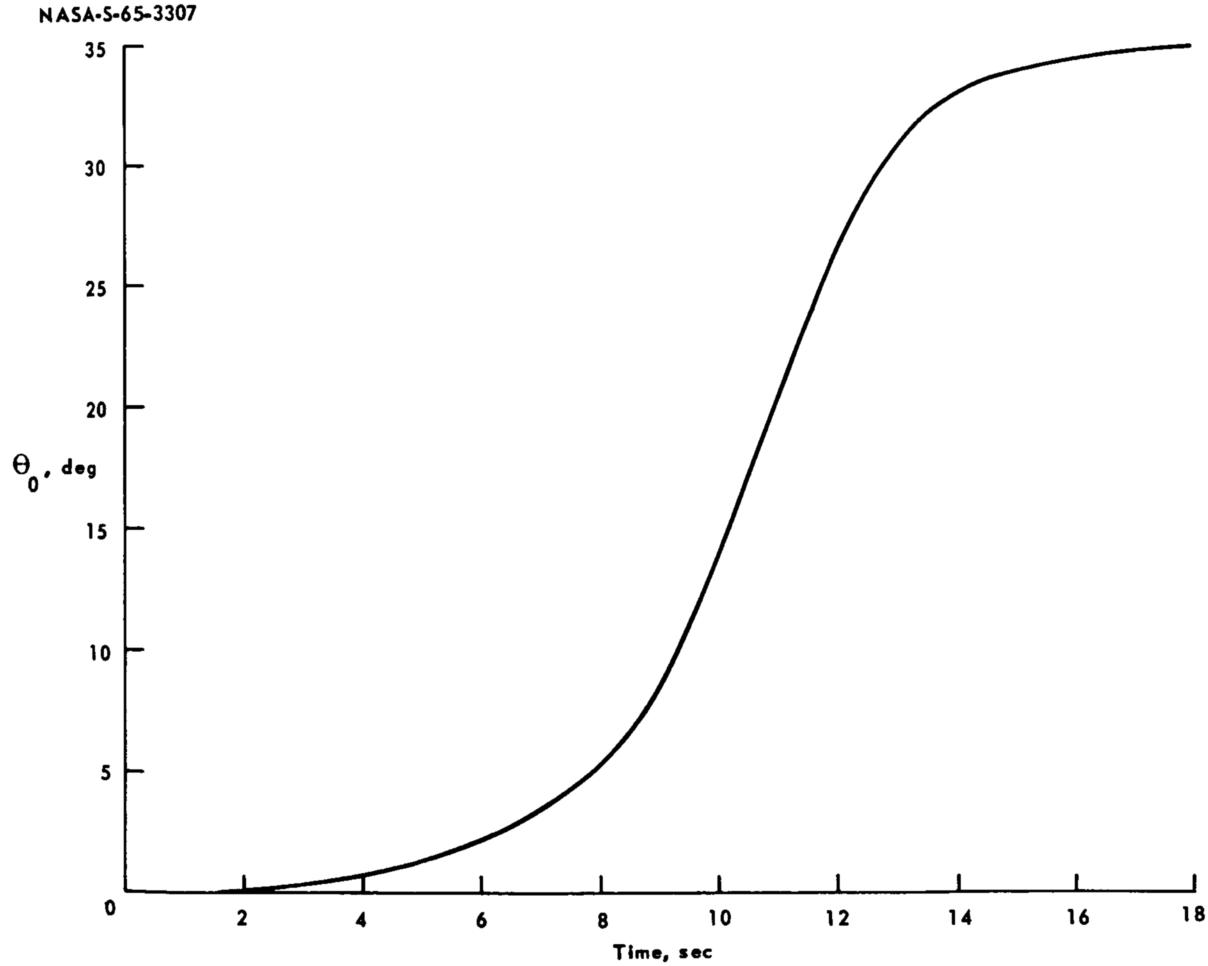

Figure 4. - Oscillation-amplitude time history for case B, $C_{m_q} + C_{m_{\dot{\alpha}}} = 2 - 6.55 \times 10^{-3} \theta^2$.

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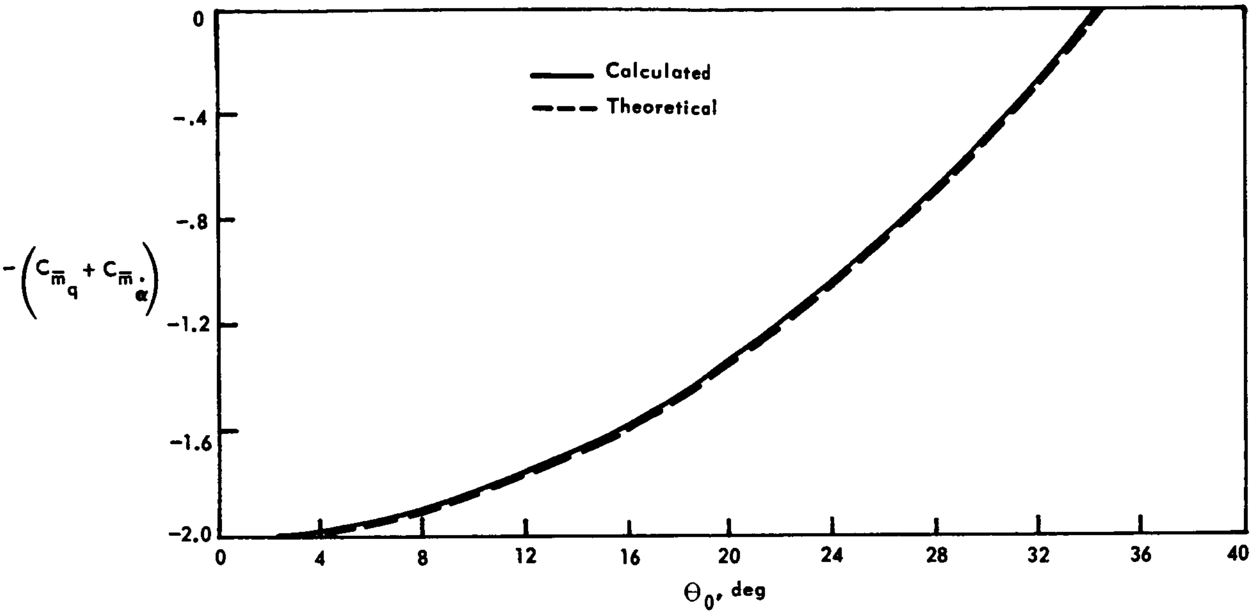

Figure 5. - Variation of $C_{\bar{m}_q} + C_{\bar{m}_{\dot{\alpha}}}$ with Θ_0 for case B.

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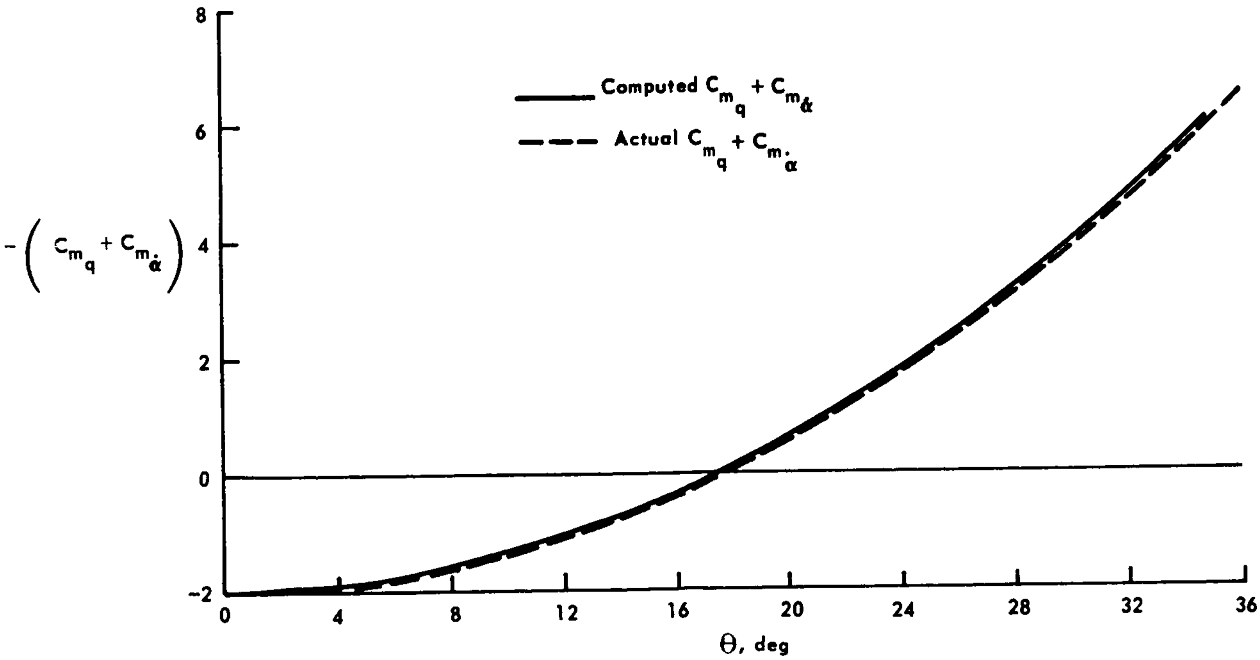


Figure 6. - Variation of $C_{m_q} + C_{m_{\dot{\alpha}}}$ with Θ for case B.

One of the important applications of method 1 is the construction of an oscillation-amplitude time history when $C_{m_q} + C_{m_{\dot{\alpha}}}$ is known by numerical integration of equation (16) rather than numerically integrating the entire equation of motion. First, $C_{m_q} + C_{m_{\dot{\alpha}}}$ is converted to $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$ by method 1. Then, by using equation (16), the oscillation time history can be constructed with small time steps so that both θ_0 and $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$ are considered to be constant. This application is illustrated for case B where:

$$C_{m_q} + C_{m_{\dot{\alpha}}} = 2.0 - 6.55 \times 10^{-3}\ \theta^2$$

By using method 1

$$C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}} = 2.0 - \left(\frac{1}{4}\right) 6.55 \times 10^{-3}\ \theta_0^2$$

This is shown in figure 5. Substituting this value in equation (16)

$$\theta_{0,2} = \theta_{0,1}\, e^{\frac{\Delta t}{2I} \frac{q_\infty SD^2}{2V}\left[2 - \left(\frac{1}{4}\right) 6.55 \times 10^{-3}\ \theta_{0,1}^2\right]}$$

The time step was altered so that θ_0 changed approximately 1° each step, and the oscillation-amplitude time history obtained is shown in figure 7. The small differences between the actual and constructed time histories are attributed to the assumption that $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$ is constant over each time step. A limit cycle of 35° is also shown in this figure. The amplitude of the limit cycle can also be found by solving for the roots of $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$

$$C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}} = \frac{q_\infty SD^2}{2V}\left[2 - \left(\frac{1}{4}\right) 6.55 \times 10^{-3}\ \theta_0^2\right] = 0$$

$$\theta_0^2 = \frac{2}{\frac{1}{4}\left(6.55 \times 10^{-3}\right)}$$

$$\theta_0 = \pm 35°$$

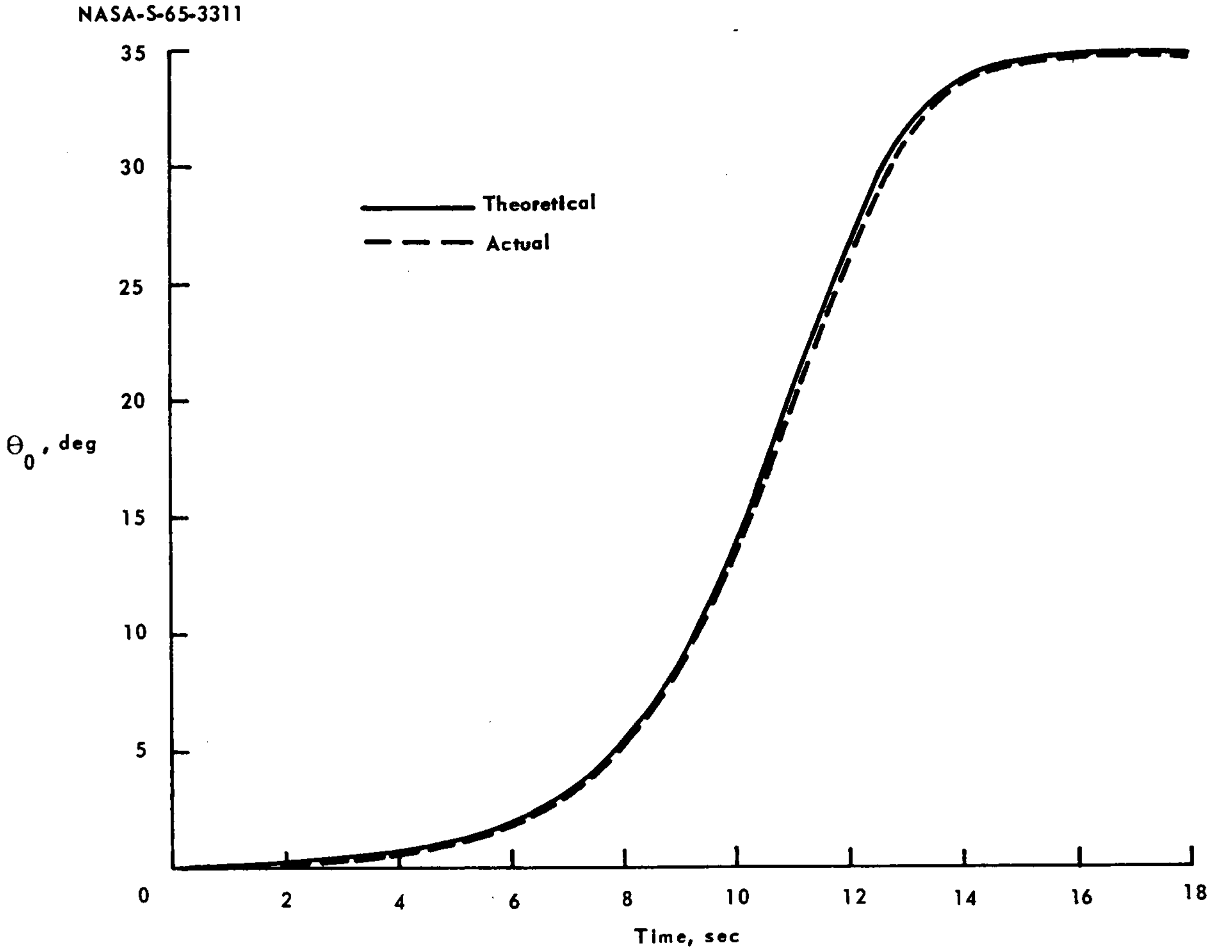


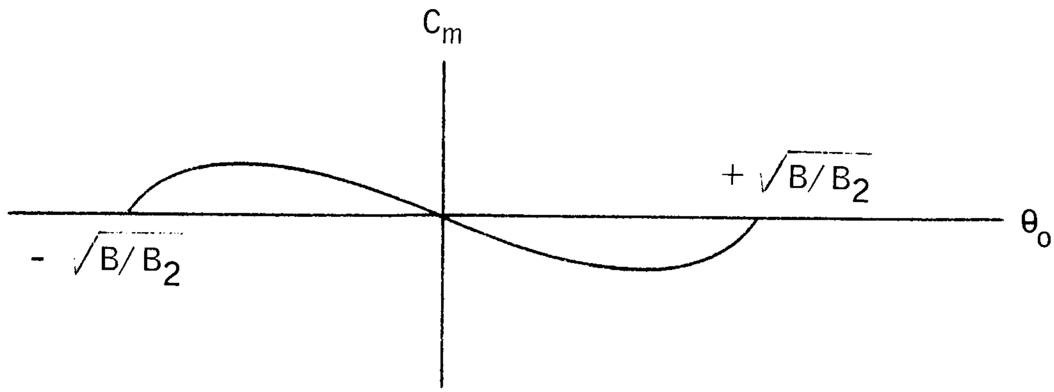
Figure 7. - Oscillation-amplitude time history constructed from given $C_{m_q} + C_{m_{\dot{\alpha}}}$, case B.

Effect of a nonlinear pitching moment.- In the development of the theory in this paper, only linear pitching moments were considered. However, except for small oscillations, the pitching moment for many vehicles is nonlinear with amplitude. To determine the usefulness of this theory as applied to the nonlinear pitching moment, the amplitude of a predicted limit cycle was compared with an actual limit cycle, since this was believed to be the most severe limitation. The nonlinear pitching-moment coefficient was simulated in the single degree-of-freedom computer program by

$$C_m = B\theta - B_2\theta^3$$

which is shown in sketch (b) where unstable trim is denoted as

$$\theta_{ut} = \pm\sqrt{B/B_2}$$

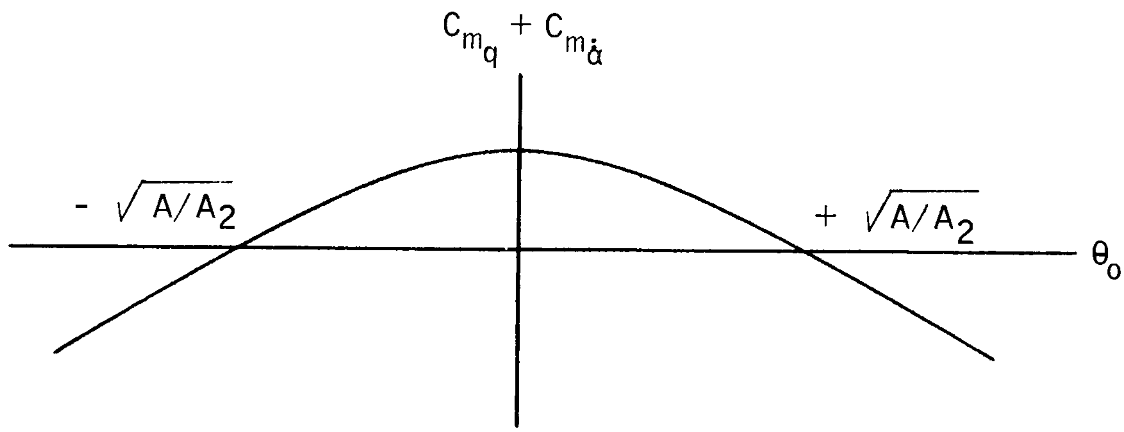


Sketch (b)

The damping coefficient was simulated in the computer program by

$$C_{m_q} + C_{m_{\dot{\alpha}}} = A - A_2\theta^2$$

as shown in sketch (c)

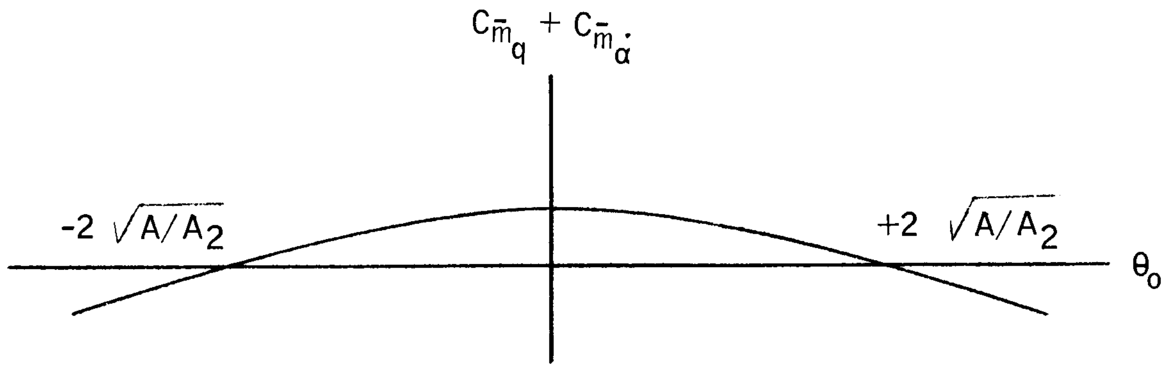


Sketch (c)

By using the method of this paper, $C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}}$ was found to be

$$C_{\overline{m}_q} + C_{\overline{m}_{\dot{\alpha}}} = A - \frac{A_2}{4}\theta_0^2$$

as shown in sketch (d) where $\pm 2\sqrt{A/A_2}$ denotes the amplitude of the theoretical limit cycle θ_{lc}. Various values of A were put into the computer program to vary the amplitude of the actual limit cycle θ^*_{lc} from 0 to the point at which tumbling occurred. The values θ^*_{lc} and θ_{lc} were nondimensionalized by dividing by θ_{ut}.



Sketch (d)

From figure 8, it can be assumed that the nonlinear pitching moment would cause no appreciable error in the solution for the amplitude of the limit cycle up to values of $0.7\ \theta_{lc}/\theta_{ut}$. From $0.7\ \theta_{lc}/\theta_{ut}$ to $0.9\ \theta_{lc}/\theta_{ut}$, the error

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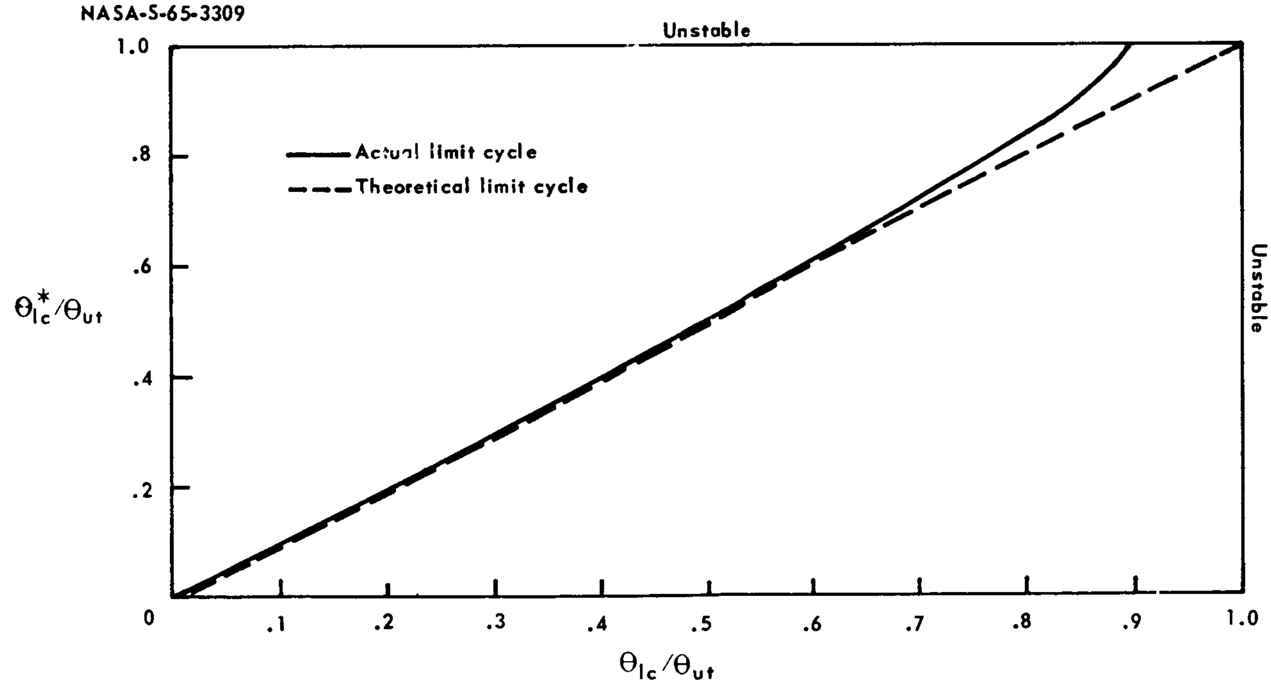

Figure 8. - Comparison of actual limit cycle with theoretical limit cycle.

increases; and for values above $0.9\ \theta_{lc}/\theta_{ut}$, the linear approximation is not usable since it predicts a limit cycle when, actually, tumbling occurs.

CONCLUDING REMARKS

The aerodynamic pitch damping coefficient as a function of instantaneous angular displacement is related with the aerodynamic pitch damping coefficient as a function of oscillation amplitude by a method in which the Kryloff-Bogoliuboff equivalent linearization technique is used. Power series expressions for the functions are developed and the like coefficients of the series are equated to provide the relationship. It is assumed that the damping per cycle is small, the damping coefficient is symmetrical, and the pitching moment is linear. Comparisons with numerical integration results show no appreciable error. The effect of a nonlinear pitching moment is considered for a representative case and is found to be small, except when approaching unstable trim.

Manned Spacecraft Center
National Aeronautics and Space Administration
Houston, Texas, March 3, 1965

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